



Department
for Education

Areas of numeracy covered by the professional skills test

December 2014

Contents

Averages	4
Mean, median and mode	4
Mean	4
Median	4
Mode	5
Avoiding common errors	8
Bar charts	9
Worked examples	11
Box and whisker diagram	13
Worked examples	16
Conversions	20
Cumulative frequency	22
Worked examples	24
Avoiding common errors	28
Formulae	29
Worked examples	30
Avoiding common errors	32
Fractions, decimals and percentages	33
Converting fractions to decimals and vice versa	33
Converting a fraction to a decimal	33
Converting a decimal to a fraction	33
Converting fractions and decimals to percentages and vice versa	34
Converting a percentage to a fraction	34
Converting a decimal to a percentage	34
Converting a fraction to a percentage	35
Simple percentage calculations	36
Using percentages to compare data	37
Worked examples	38
Avoiding common errors	40
Line graphs	41
Worked examples	42

Avoiding common errors	43
Pie-charts	44
Worked examples	45
Using pie charts to compare data	46
Worked example	46
Proportion	48
Worked examples	48
Avoiding common errors	49
Range	50
Worked examples	50
Ratio	53
Avoiding common errors	54
Scatter graphs	55
Making comparisons	55
Worked examples	57
Avoiding common errors	58
Time	59
Avoiding common errors	60
Two way tables	61
Avoiding common errors	63
Weighting	64

Averages

An average is a single number which is used to represent a group of values collected for a particular purpose.

Mean, median and mode

Mean

The mean is the most commonly used 'average' and the 1 often referred to in everyday conversation, eg the average wage. The mean is usually used when the data involved is fairly evenly spread, and there are no exceptional cases that are much higher or lower than the rest. If a few exceptional results exist, the mean may give a misleading impression, because it takes account of all the data given. It is found by adding together all the data values in a set of data and then dividing this total by the number of values in the set.

Example

In a test, a group of 11 pupils scored the following marks:

5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5

The mean mark is found by adding together all the marks and dividing the total by the number of pupils.

$$(5+10+3+4+4+8+4+3+11+9+5)\div 11$$

$$= 66 \div 11$$

$$= 6$$

Therefore the mean mark is 6.

Median

The median is the middle value of a set of data when placed in order. It can be found with little or no calculation. The median is particularly useful when the data has a wide spread, as the middle value is not affected by exceptional cases. To find the median age of a group of 11 pupils it is only necessary to arrange their ages in order, and the age of the middle child (the sixth child) is the median. This means that for the median there are as many values in the dataset above the median as there are below it. The median for any data can also be found by drawing a cumulative frequency graph. See the page on cumulative frequency for more information.

Example

The median mark is the middle value in the group of marks when arranged in order of size.

In order of size the marks are:

3	3	4	4	4	5	5	8	9	10	11
					↑					
Middle value (Median)										

There are 11 numbers in this set of data. The sixth number is the middle value or median. In this example the median is 5.

Note: When there is an even number of values, the median is found half way between the 2 middle values.

For example, if the test results for a particular pupil are: 12, 13, 16, 17, 21 and 25, then 16 and 17 are the middle values. The median is the mean of the 2 values, or half way between the 2 values. That is $(16 + 17)/2$. The median in this example is 16.5.

Mode

The mode is the most frequently occurring value in a set of data. For example shoe manufacturers are not interested in the mean or median value of shoe sizes but they may want to know the size most frequently sold.

Example

The mode is the value which occurs most often in the set of marks.

For the set of test marks:

5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5

The score which occurs most often is 4 with 3 pupils scoring 4 marks, so the modal mark, or mode, is 4.

Worked examples

Example 1

The table below records the amount of time, rounded to the nearest quarter of an hour, each member of a class spends on homework in hours.

Time spent on homework during the week ending 24/1/2003

Pupil	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Hours	$2\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	3	$1\frac{3}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	$1\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{1}{2}$	2	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{3}{4}$

What is the mean amount of time pupils spend on homework?

To find the mean, first add all the values together. The answer is 30.

Then divide by the number of items, in this case, 15.

So the mean is $30/15 = 2$.

Therefore the mean amount of time spent on homework during the week is 2 hours.

The teacher may also be interested in the most common amount of time spent by pupils, the mode:

Time in hours	$\frac{3}{4}$	$1\frac{1}{2}$	1	2	$2\frac{1}{2}$	$2\frac{3}{4}$	3
Number of pupils=15	1	4	3	1	3	2	1

The amount of time pupils spent varies a good deal. However, 4 pupils spent 1.5 hours.

Therefore the mode is 1.5 hours.

Example 2

A teacher sets a test for 20 pupils that is marked out of a possible total of 20 marks. The teacher wants to know the median mark for the test.

Ordering the test scores gives:

19	19	19	18	18	17	16	15	15	15	14	14	14	12	12	11	10	10	2	1
									↑	↑									

As there are 20 marks there will be 2 middle marks, the 10th and 11th. These are: 15 marks and 14 marks.

The median is half way between them, or the mean of the 2 marks.

So the median is 14.5 marks.

Example 3

The set of data below is the GCSE business studies grade results for a group of 70 pupils.

Business studies GCSE results 1999

A	D	D	U	C	B	E
G	A*	D	B	F	A	G
F	C	C	E	C	C	D
D	F	F	C	C	D	U
D	D	A	U	A*	C	D
E	D	G	D	F	C	D
B	E	B	U	G	B	C
C	E	F	C	C	F	F
C	F	E	A*	B	B	G
D	E	E	G	B	C	G

The teacher wants to know what the average performance is and chooses the mode as the most appropriate average, that is, the grade that the most pupils achieve.

To find the mode the data needs to be sorted. The number of times each grade occurs is counted and entered as below:

Grade	A*	A	B	C	D	E	F	G	U
Number of pupils achieving grade	3	3	8	15	13	8	9	7	4

The grade that occurs most frequently is C. So C is the modal grade.

The teacher might also be interested in the average (mean) points score. If points are attributed to grades as follows:

Grade	A*	A	B	C	D	E	F	G	U
Points	8	7	6	5	4	3	2	1	0

The mean can be calculated by:

- multiplying the points score by the number of pupils at each grade
- finding the total
- dividing by the number of pupils (70):

$$(3 \times 8 + 3 \times 7 + 8 \times 6 + 15 \times 5 + 13 \times 4 + 8 \times 3 + 9 \times 2 + 7 \times 1 + 4 \times 0) \div 70$$

$$= 269 \div 70$$

$$= 3.8 \text{ to 1 d.p.}$$

The average (mean) points score is 3.8.

The nearest grade to 3.8 points is D.

This example shows that the mean and the mode do not necessarily give the same result. Both can be useful in this case, but are giving the teacher different information.

One tells the teacher the most common grade, while the other gives an average (mean) points score which might be useful for comparison with other subjects. The fact that the mean is less than the mode indicates that the results as a whole are likely to be spread out among the lower grades as shown in the second table above.

Avoiding common errors

Whenever the word 'average' is used, make sure you understand which average is being referred to in the context of the set of data: is it the mean, median or mode?

When finding the median, make sure that the data is arranged in ascending or descending order.

Bar charts

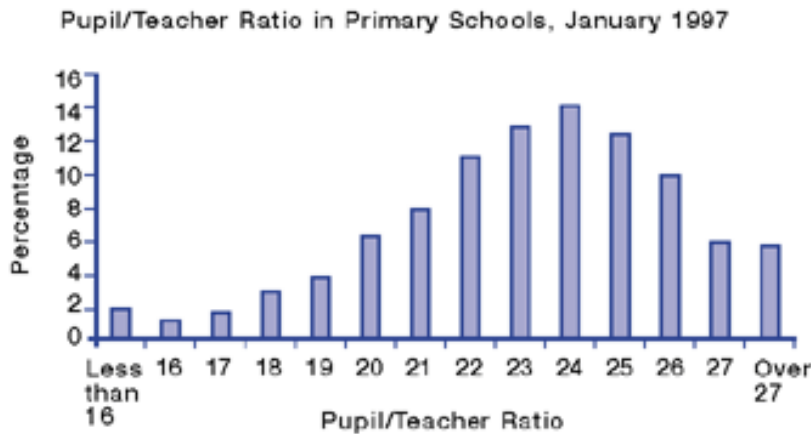
A bar chart shows items represented as vertical or horizontal bars. The length of each bar shows the number of times the item occurs.

Example 1

A survey of primary schools was carried out in January 1997. The number of pupils and all teachers, including the head teacher, was recorded for each primary school surveyed. The pupil to teacher ratio (PTR) was calculated for each school. The data was recorded in the table shown below.

Pupil teacher ratio (PTR)	<16	16	17	18	19	20	21	22	23	24	25	26	27	>27
% of schools with this PTR	2.0	1.2	1.9	2.9	4.0	6.5	8.0	11.2	13	14.3	12.8	10.3	6	5.9

A bar chart was produced to help the interpretation of the data.



The bar chart shows the data in a visual form.

For each PTR value, a vertical bar has been drawn. The top of the bar represents the corresponding percentage value from the table. For example, the bar for a PTR less than 16 has a height of 2 on the vertical axis, representing 2%, and the bar for a PTR of 23 is represented by 13% on the vertical axis.

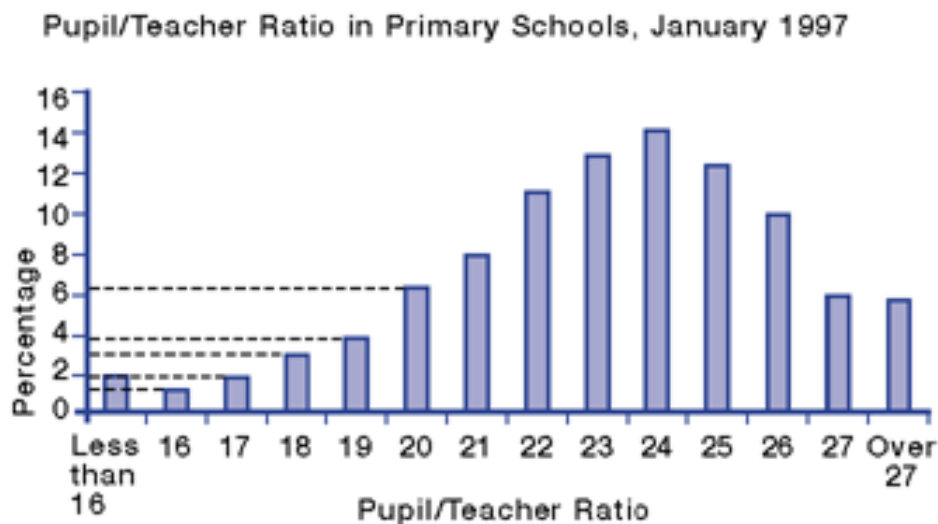
It is often easier to see the pattern in data when it is displayed as a graph rather than in a table. This bar chart shows that most schools have a PTR between 22 and 26 and the most common PTR is 24.

Bar charts are often used in documents containing data, rather than tables. Information must be obtained by using the charts.

Example 2

Use the bar chart to find what percentage of schools have a PTR of 20 or less.

This question can be solved by locating the bars for PTRs of 20 or less, (ie the bars for less than 16, up to and including 20). As the values must be read from the bar chart, some values are likely to be approximate because they cannot be read accurately.



$$2 + 1.2 + 1.9 + 3 + 4 + 6.5$$

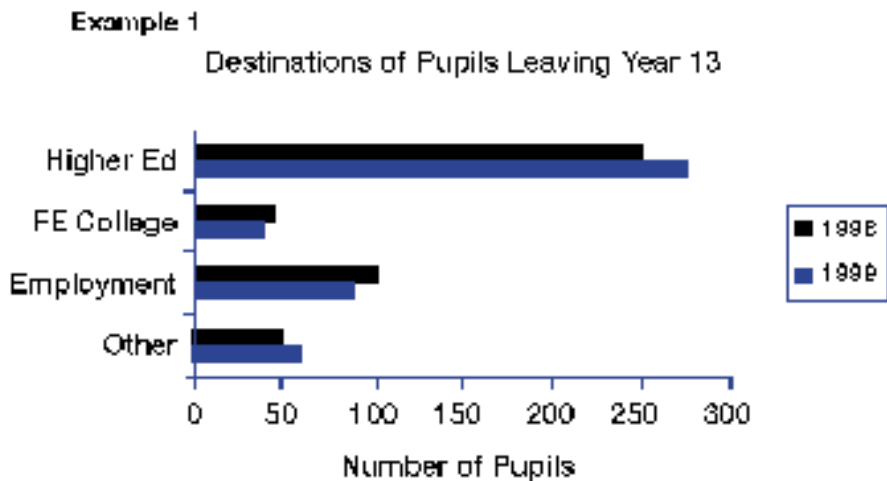
$$= 18.6\%$$

So, approximately 18.6% of schools have a PTR of 20 or less.

Worked examples

Example 1

How many pupils entered employment in 1998?

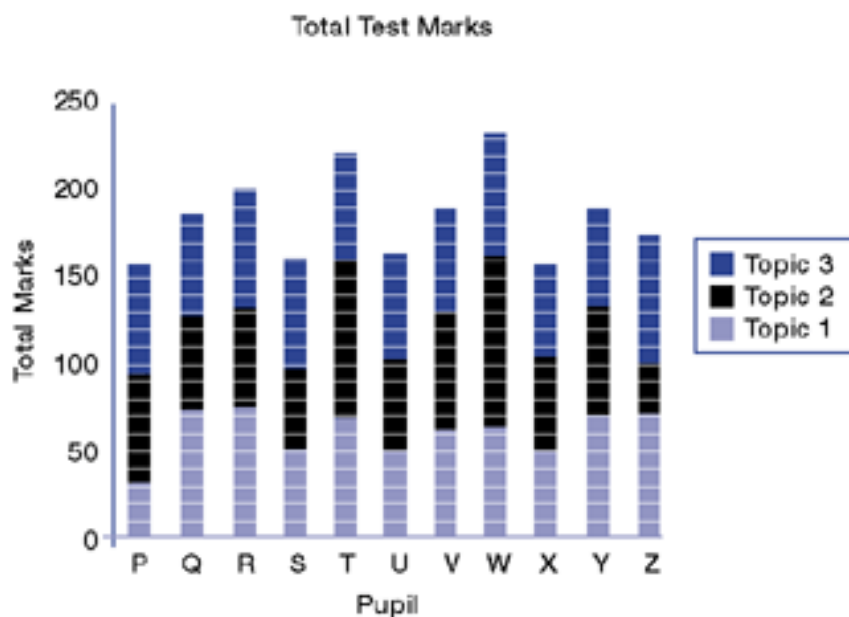


Look at the key to find the appropriate shading for 1998.

Find 'employment' and the end of the shaded bar for 1998. Find the corresponding number on the horizontal 'number of pupils' axis.

So, 100 pupils entered employment in 1998.

Example 2



This is a composite bar chart which shows the marks achieved by 11 pupils in 3 tests. Each test covers a different topic from the programme of study. The test result for each topic is shaded and forms part of each bar as shown on the chart.

Which of the following statements are true?

1. Pupil S had the lowest total marks for all 3 tests
2. Pupil V had the highest mark in topic 1
3. None of the pupils scored 200 or more marks
4. Pupil W had the highest score on topic 2

Look at each statement in turn and find the information on the graph. Check whether or not the statement is true.

1. The lowest total mark is found by looking at the tops of all the bars. Pupil S had 160 marks. Pupil P had the lowest overall mark (157). So this statement is not true.
2. Pupil V had 60 marks in topic 1, while Pupil R had the highest mark (74) in topic 1 so this statement is not true.
3. Pupils R, T and W scored 200 or more marks so this statement is not true.
4. Scores on topic 2 can be found by looking at the height of the dark shaded sections of each of the bars or by finding the difference between value on the vertical axis at the top of the dark shaded section of the bar and the value on the vertical axis at the bottom of the dark shaded section of the bar. Pupils T and W scored highly on topic 2. Comparing these on the chart it can be seen that pupil W has the highest score so this statement is true.

The only statement that is true is statement 4.

Box and whisker diagram

A box and whisker diagram illustrates the spread of a set of data. It also displays the upper quartile, lower quartile and inter-quartile range of the data set.

A quartile is any 1 of the values which divide the data set into 4 equal parts, so each part represents a quarter of the sample. The upper quartile represents the highest 25% of the data. It can be considered as the median of the upper half of the values in the set.

The lower quartile represents the lowest 25% of the data. It can be considered as the median of the lower half of all the values in the set. The inter-quartile range is the difference in value between the upper quartile and the lower quartile values. The median is the middle value, half of the data set is below and half is above.

Example

You can draw a box and whisker diagram for the results below which were obtained by a year 6 class in an English test marked out of 20.

Pupil A - 14

Pupil B - 13

Pupil C - 3

Pupil D - 7

Pupil E - 9

Pupil F - 12

Pupil G - 17

Pupil H - 4

Pupil I - 9

Pupil J - 10

Pupil K - 18

Pupil L - 16

There are 12 scores. You must place these in order as follows 3, 4, 7, 9, 9, 10, 12, 13, 14, 16, 17, 18. The range is $18 - 3 = 15$.

There is an even number of values so the median is the midway between 10 and 12. The median has the value 11.

The upper quartile is the median value for 12, 13, 14, 16, 17, 18 and so is midway between 14 and 16. The upper quartile is 15. So 25% of all the pupils score above 15 and 75% score less than 15.

The lower quartile is the median value for 3, 4, 7, 9, 9, 10 and so is midway between 7 and 9. The lower quartile is 8. So 25% of all the pupils score less than 8 and 75% score more than 8.

Had there been an odd number of values, the median would be the middle value, but in determining the lower and upper quartiles the median value is then ignored. For example, if the values are:

2, 3, 5, 7, 9, 13, 15, 18, 19, 20, 22, 23

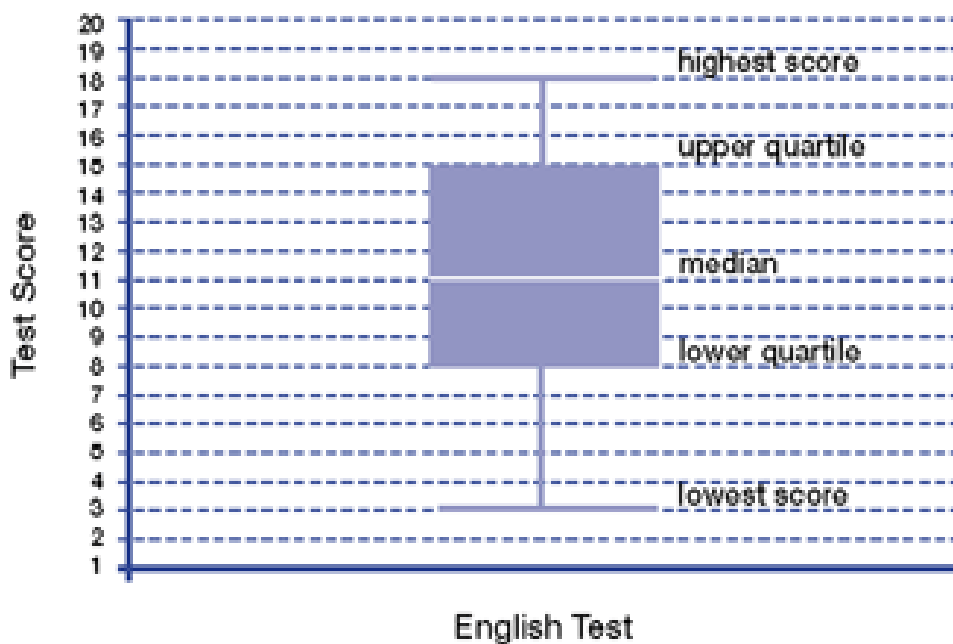
The median value is 14.

A box and whisker diagram can be used to display this information. The table below shows the results of an English test, constructed for presentation as a box and whisker diagram.

English test scores out of 20

Lowest test score	3
Highest test score	18
Lower quartile	8
Upper quartile	15

The data in the table is represented by a box and whisker diagram as follows:



A box and whisker diagram shows at a glance the range of scores of the middle 50% of pupils (the box) and the total range of all the scores (the whiskers).

A box and whisker diagram shows at a glance the range of scores of the middle 50% of pupils (the box) and the total range of all the scores (the whiskers).

The 2 whiskers show the highest and lowest values from which the range of the data set can be calculated. For this data set, the range is given by $18 - 3 = 15$.

The box gives the range of the middle 50% normally called the inter-quartile range. It represents the range of the scores between 25% (the lower quartile) and 75% (the upper quartile), hence the middle 50%. In this example, the inter-quartile range is given by $15 - 8 = 7$. Comparisons between different sets of data (eg different test scores) can be made by plotting a box and whisker diagram for each set of scores on the same graph.

Box and whisker plots can be constructed using cumulative frequency curves or cumulative percentage curves as these curves can be used to show the quartiles and the median. You should also read the section on cumulative frequency graphs.

Worked examples

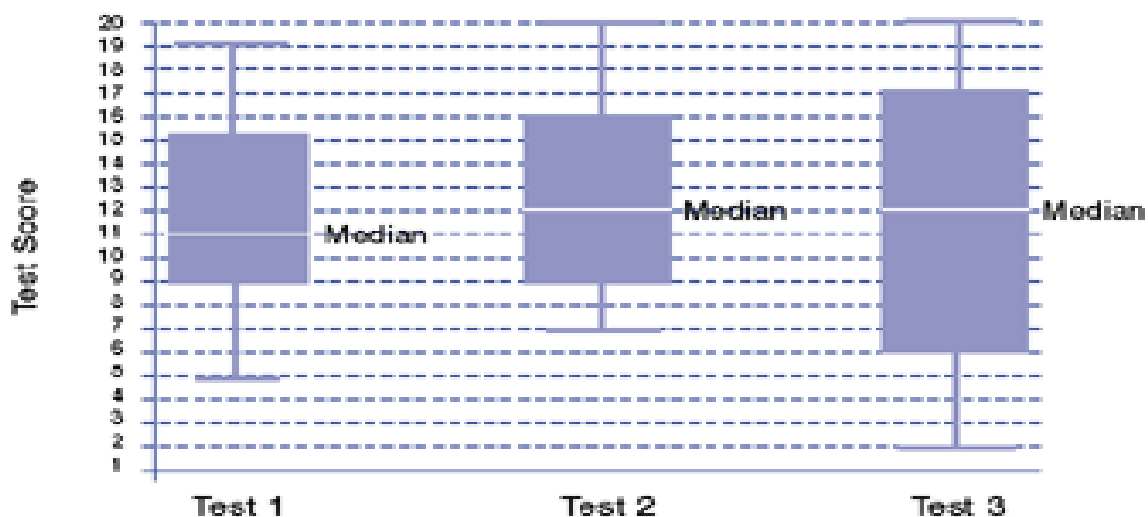
Example 1

A year 9 class completed 3 science tests. The scores for the 3 tests are presented in the table below. Display this data using a box and whisker diagram.

Year 9 science test scores - marks out of 20

	Test 1	Test 2	Test 3
Lowest score	5	7	2
Highest score	19	20	20
Lower quartile	9	9	6
Upper quartile	15	16	17

The box and whisker diagrams for each test are shown below:



The diagrams show that in test 3 pupils achieved a wider spread of results (from 2 to 20 with a range of 18) than in tests 1 and 2. This could lead teachers to ask questions about why pupils' performance ranged more widely on this test than on the others. In the diagram for test 1, the 2 'whiskers' are the same length, so the upper and lower quartile ranges are the same.

Example 2

The scores for 3 tests completed by a year 5 group are:

	Test 1 (score out of 20)	Test 2 (score out of 50)	Test 3 (score out of 100)
Lowest score	5	12	20
Highest score	19	43	75
Lower quartile	9	19	35
Upper quartile	15	35	70

Which of the following statements are true?

1. The highest and lowest marks for the year group declined over the 3 tests
2. There was no change in the year group's performance over the 3 tests
3. Marks for the year group improved over the 3 tests

All the tests have different maximum scores and will be easier to compare if converted into percentages. Box and whisker diagrams can be drawn on the same scale and comparisons made between the tests.

Test 1 scores are out of 20 and need to be converted so that they are out of 100, by multiplying top and bottom of the fraction by 5 (because $5 \times 20 = 100$).

$\frac{5}{20}$	=	$\frac{25}{100}$	so	$\frac{5}{20}$	= 25%
$\frac{19}{20}$	=	$\frac{19 \times 5}{100}$	so	$\frac{19}{20}$	= 95%
$\frac{9}{20}$	=	$\frac{9 \times 5}{100}$	so	$\frac{9}{20}$	= 45%
$\frac{15}{20}$	=	$\frac{15 \times 5}{100}$	so	$\frac{15}{20}$	= 75%

Test 2 scores are out of 50, so can be converted by doubling each score.

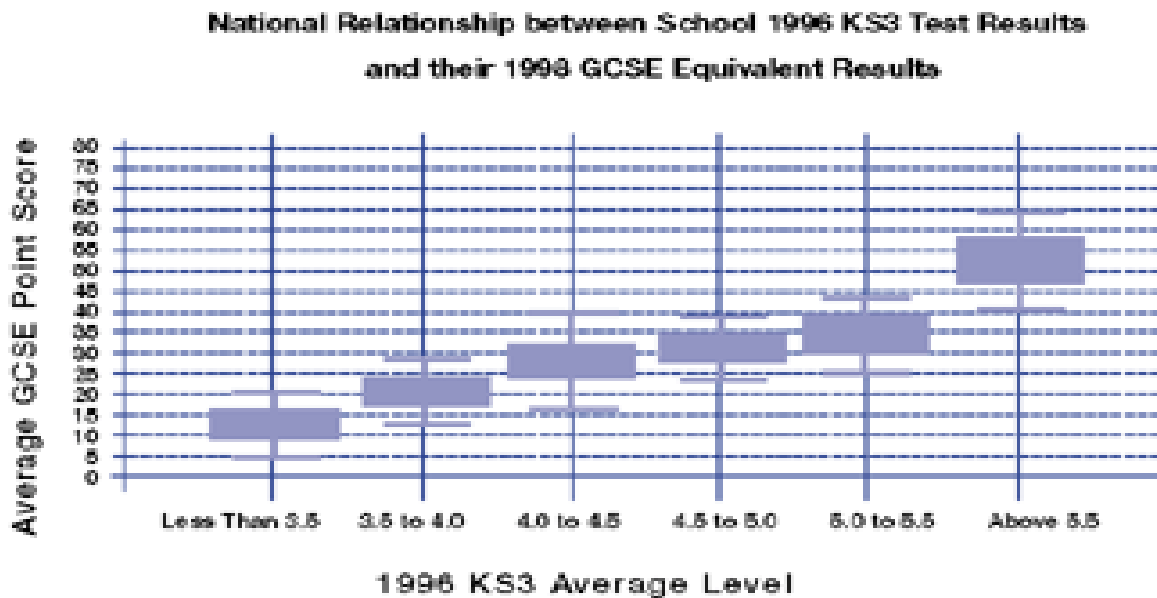
$\frac{12}{50}$	=	$\frac{24}{100}$	= 24%
$\frac{43}{50}$	=	$\frac{86}{100}$	= 86%
$\frac{19}{50}$	=	$\frac{38}{100}$	= 38%
$\frac{35}{50}$	=	$\frac{70}{100}$	= 70%

Test 3 scores are already in percentages, because they are already given out of 100.

The figures show that both the highest and lowest scores have fallen from test 1 to test 2, and from test 2 to test 3. As statement 1 is true, statements 2 and 3 must be false.

Example 3

National relationship between school 1996 key stage 3 test results and their 1998 GCSE equivalent results



Note: for clarity, the median for each test is not indicated.

Which of the following statements are true for schools in the band 'scoring less than level 3.5'?

1. The lowest average GCSE point score was 5
2. The highest GCSE point score achieved by the bottom 25% of schools was 27
3. The lowest average GCSE point score achieved by the top 25% of schools was 15

To answer the question, consider if each statement is true. We only need to look at the first box and whisker diagram, which is for an average level of less than 3.5.

Statement 1

Look at the diagram and select the lower whisker.

Read horizontally from the lower whisker to the vertical scale.

The average GCSE score is 5.

The statement is therefore true.

Statement 2

Look at the diagram and look at the bottom of the box.

Read across to the vertical scale.

The average GCSE point score is about 8.

The statement is therefore not true.

Statement 3

Look at the graph and look at the top of the box.

Read across to the vertical scale.

The average GCSE score is 15.

The statement is therefore true.

Therefore statements 1 and 3 are true.

Conversions

Conversion involves changing information from 1 unit of measurement to another. For example, converting euros to pounds sterling (£), distances in kilometres to miles, or weights from pounds to kilograms.

Example 1

On a school trip to Spain the entrance fee to a museum for a group of pupils is 64 euros. If the exchange rate is 1.6 euros = £1, what is the group entrance fee in pounds sterling?

Divide the total entrance cost in euros by the numbers of euros equivalent to 1 pound:

$$64 \div 1.6$$

This calculation is made easier if you multiply both numbers by 10:

$$640 \div 16 = 40$$

So the group entrance fee in pounds sterling is £40.

Example 2

A teacher led a group of pupils on a camping trip to France. The teacher wanted to arrange a cycle ride of approximately 8 miles for the pupils to a local village and back. The signpost at their campsite showed the distance to the following villages:

La Pet 7km

Fouin 6km

Travin 5km

Taking 5 miles to be equivalent to 8 kilometres, which of the three villages should the teacher choose as the destination for the cycle ride?

The return distance from the campsite to La Pet is $2 \times 7\text{km} = 14\text{km}$

Convert this to miles by dividing by 8 and multiplying by 5:

$$14 \div 8 \times 5 = 8.75 \text{ miles}$$

The return distance from the campsite to Fouin is $2 \times 6 = 12\text{km}$

Convert this to miles by dividing by 8 and multiplying by 5:

$$12 \div 8 \times 5 = 7.5$$

The return distance from the campsite to Travin is $2 \times 5\text{km} = 10\text{km}$

Convert this to miles by dividing by 8 and multiplying by 5:

$$10 \div 8 \times 5 = 6.25 \text{ miles}$$

You can also consider that the distance to Travin is less than the distance to Fouin in kilometres. Given that the return distance from the camp to Fouin is 7.5 miles, it is clear that Travin is not the right answer.

You can see that 7.5 miles is closer to 8 miles than 8.75 miles, so Fouin is the village to choose as the destination for the cycle ride.

Example 3

For some conversions a formula may be used.

An approximate conversion from kilometres to miles is given by the formula

$$M = \frac{5}{8}K$$

where K represents the number of kilometres and M represents the number of miles. To convert 12 kilometres into miles using the formula, you can substitute as follows:

$$M = \frac{5}{8} \text{ of } 12$$

$$M = \frac{60}{8} \text{ (by multiplying 5 by 12)}$$

$$M = 7.5 \text{ (by dividing by 8)}$$

So 12 kilometres is approximately 7.5 miles.

To convert 60 miles into kilometres substitute into the formula as follows:

$$60 = \frac{5}{8} K$$

$$60 \times 8 = 5 K \text{ (multiply both sides by 8)}$$

$$480 = 5 K$$

$$K = 480 \div 5 \text{ (divide both sides by 5)}$$

$$K = 96$$

So 60 miles is approximately 96 kilometres.

Cumulative frequency

A cumulative frequency graph shows the cumulative totals of a set of values up to each of the points on the graph.

Example

A teacher arranged the marks gained by all year 10 pupils in a mathematics test in a table as shown below:

Marks	Frequency of pupils
11-20	2
21-30	11
31-40	19
41-50	36
51-60	42
61-70	31
71-80	13
81-90	6

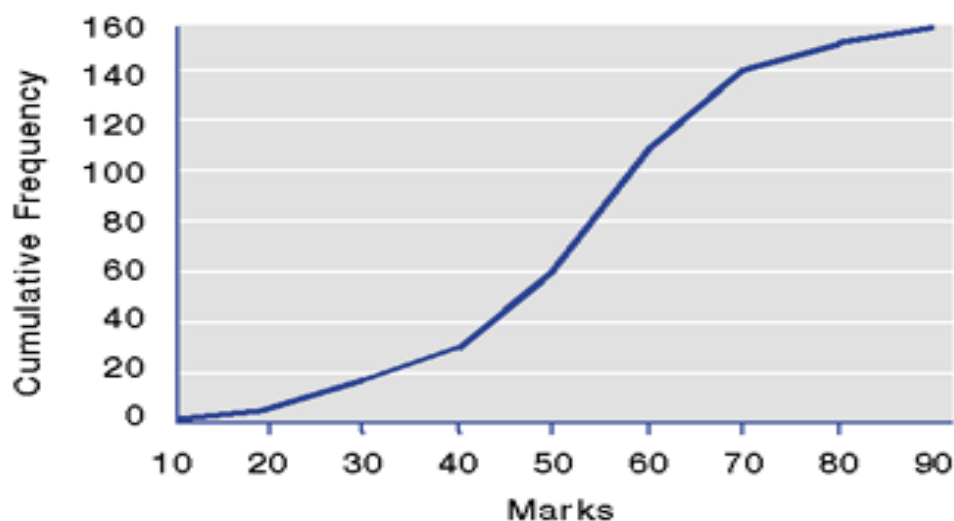
This table shows the number of pupils (called the frequency) who gained marks in the various mark bands (eg 31 to 40). For example, the number of pupils who scored between 21 and 30 marks was 11. No pupil scored fewer than 11 marks or more than 90 marks.

To create a cumulative total for the frequency of pupils in each group (called the cumulative frequency) a third column is created as shown below:

Marks	Frequency	Cumulative total	Cumulative frequency
11-20	2	2	2
21-30	11	2+11	13
31-40	19	13+19	32
41-50	36	32+36	68
51-60	42	68+42	110
61-70	31	110+31	141
71-80	13	141+13	154
81-90	6	154+6	160

The cumulative frequency column makes it easy to see at a glance that 68 pupils scored 50 marks or fewer, and that 32 pupils scored 40 marks or fewer.

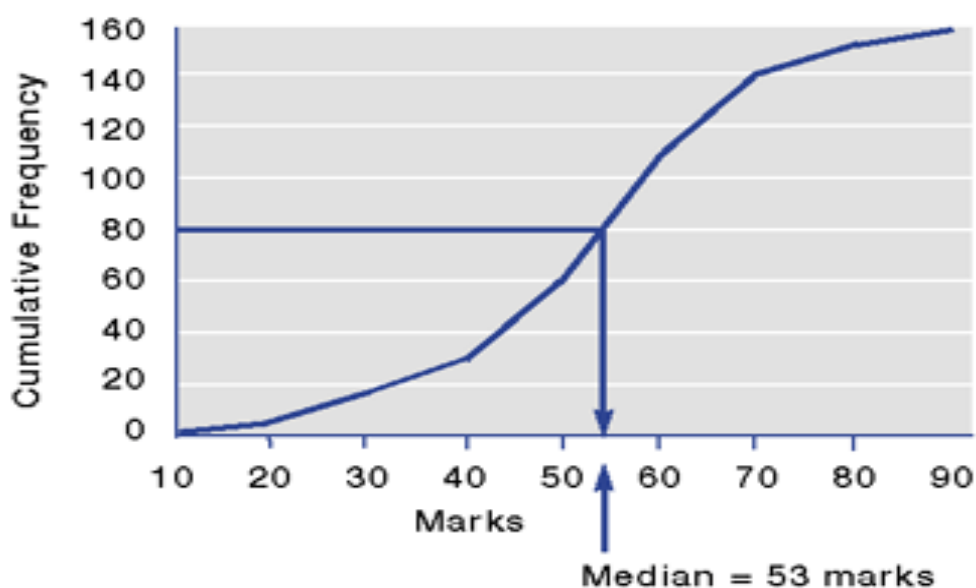
Cumulative Frequency Graph for Year 10 Mathematics Results



A cumulative frequency graph is a way of presenting information visually, which allows other information to be deduced.

For example, from the graph we can obtain the median (or middle) mark. The median is the mark which half of all pupils exceed and half do not reach. As there are 160 pupils, we need to find 80 pupils, and then draw a line across until it meets the graph. Drawing a vertical line down from this point and reading the number of marks at that point shows that the median is 53 marks.

Cumulative Frequency Graph for Year 10 Mathematics Results



It is also possible to find the upper and lower quartile marks from the graph.

In this example, the lower quartile is the mark which 1 quarter of all pupils' scores does not reach.

To find the lower quartile, find the point 1 quarter of the way up the vertical axis, which is 40 on the cumulative frequency axis. Draw a line across from the 40 mark until it meets the graph. Draw a vertical line down from this point. The lower quartile is 43 marks. Therefore a quarter of the marks lie below 43 marks.

To find the upper quartile, we need to find the point that is three quarters of the way up the vertical axis which is 120 on the cumulative frequency axis. Draw a line across from the 120 mark until it meets the graph. Draw a vertical line down from this point. An estimate of the upper quartile is 63. Therefore three-quarters of the marks lie below 63 marks.

The inter-quartile range is often used to give an idea of how widely the items of data are spread out.

The inter-quartile range is found by calculating the difference in value between the upper quartile and the lower quartile, that is:

$$\text{upper quartile value} - \text{lower quartile value.}$$

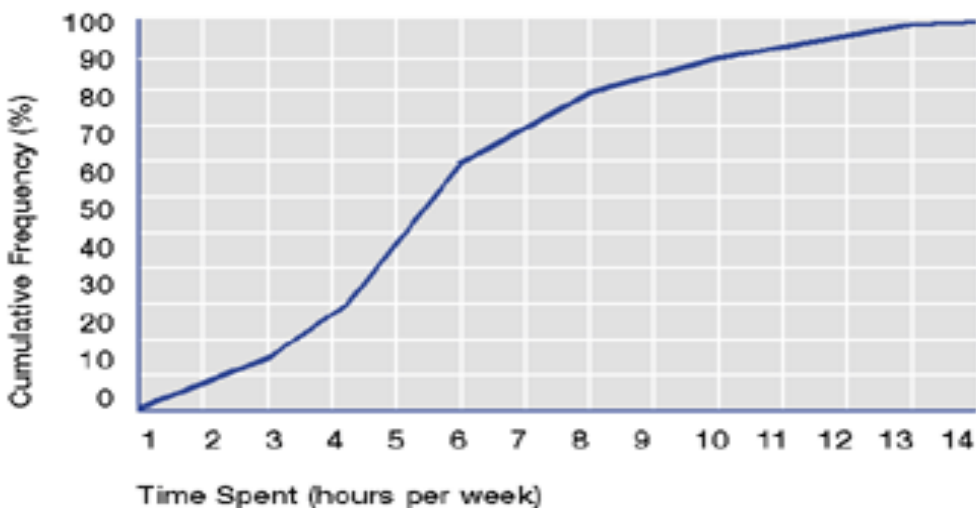
In this example, the estimate of the inter-quartile range is $63 - 43 = 20$.

The marks of the middle 50 per cent of pupils lie roughly between the lower quartile mark, 43, and the upper quartile mark, 63.

If this information was shown using a box and whisker diagram, the box would be drawn with the left-hand edge at 43 and the right-hand edge at 63.

Worked examples

Cumulative Frequency Graph for Time Spent on Homework



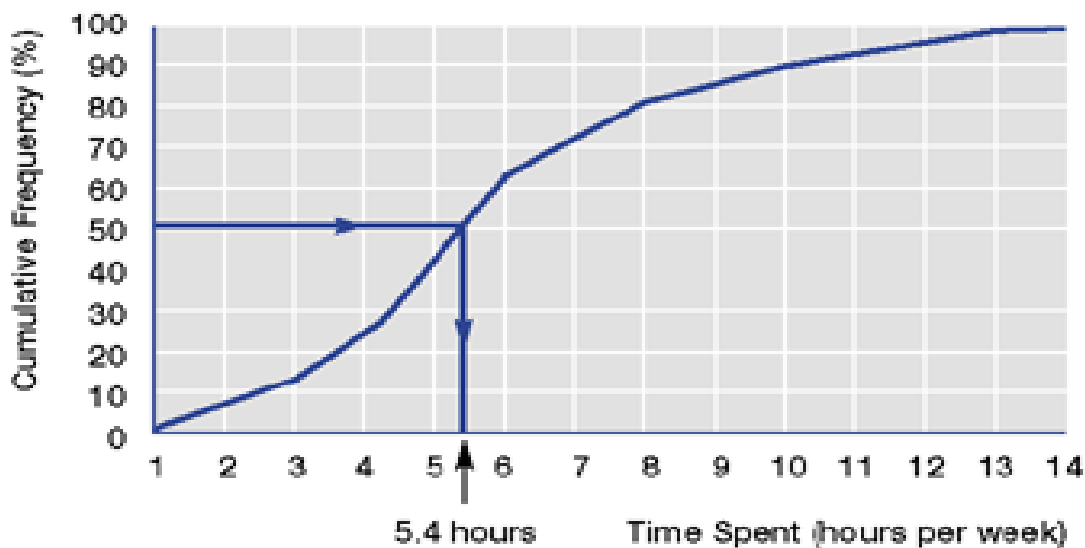
This is a cumulative frequency graph illustrating, in percentages, the amount of time secondary school pupils spend on homework.

Example 1

What is the median time that is spent on homework?

The median value is the number of hours spent on homework by the middle pupil(s). The middle value will be at 50% as the graph shows values from 0-100 %.

Cumulative Frequency Graph for Time Spent on Homework



From this point read across to the graph and draw a vertical line down to the horizontal axis, giving an estimate of the median as 5.4 hours.

The median is 5.4 hours, or 5 hours 24 minutes.

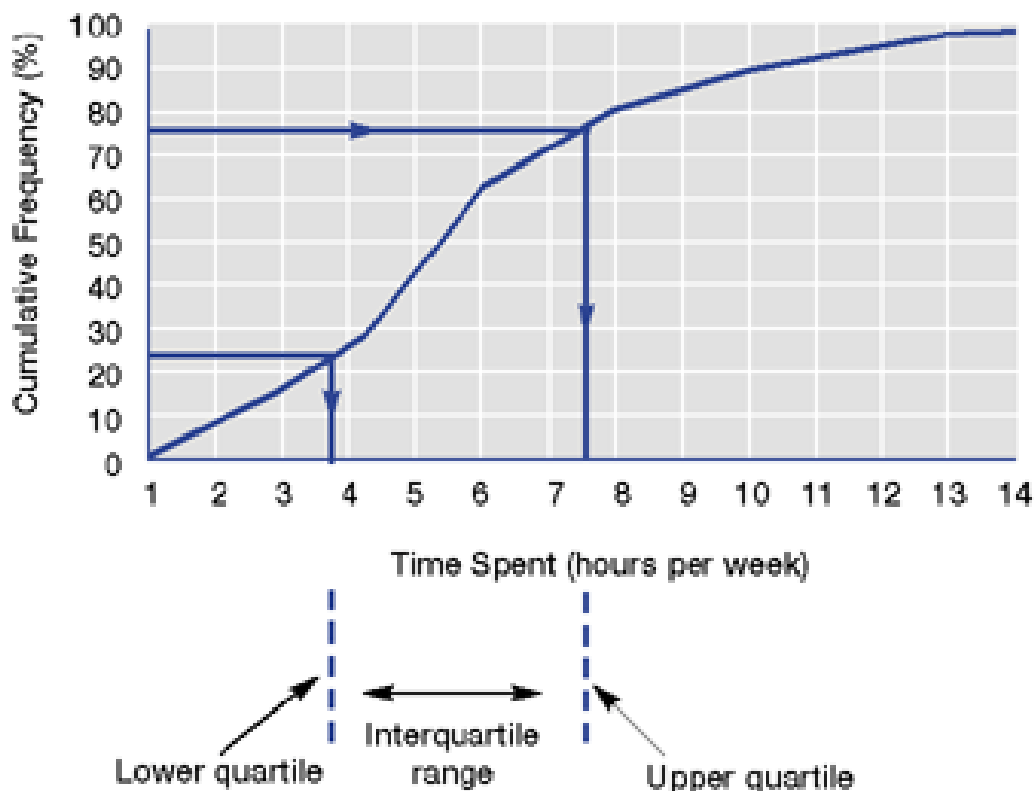
Example 2

What is the inter-quartile range?

It is necessary to first find the upper and lower quartiles.

To find the lower quartile find the number of hours of homework done by 25 % or fewer of the pupils.

Cumulative Frequency Graph for Time Spent on Homework



Reading across from 25 % and down from the graph gives an estimate of the lower quartile as just under 4 hours.

Following the same process, the upper quartile at 75 % is about 7½ hours.

From this an estimate of the inter-quartile range is found as:

$$\text{upper quartile} - \text{lower quartile}$$

$$= 7\frac{1}{2} \text{ hours}$$

So the inter-quartile range is roughly 3½ hours.

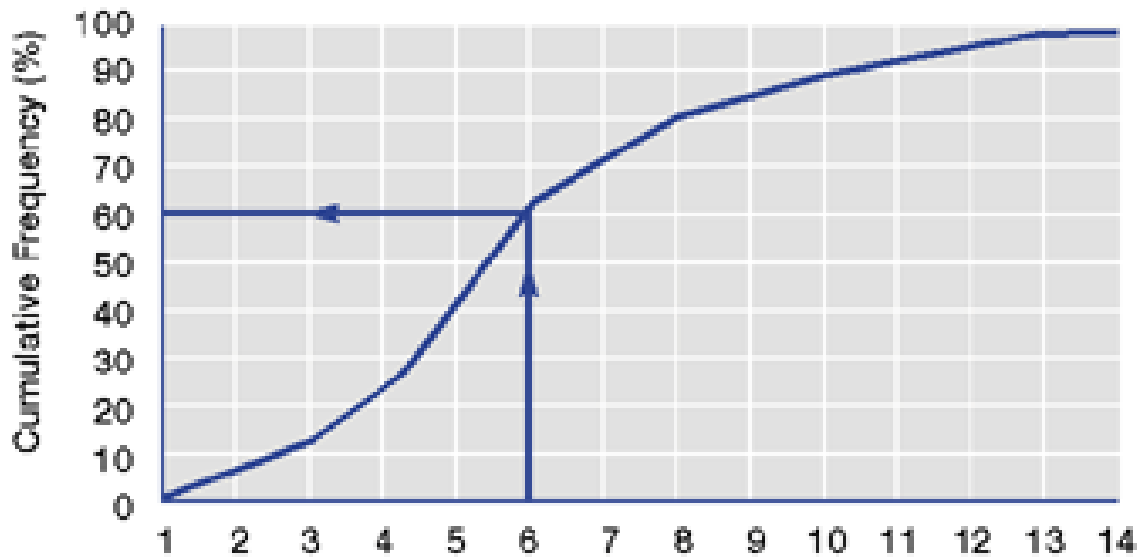
The number of hours spent on homework by the middle 50 % of the pupils ranges from 4 hours to 7½ hours.

Example 3

What percentage of pupils spends 6 hours or less on homework per week?

Find the point marked '6 hours' on the horizontal axis. Move from the axis vertically up to the curve. Move from the curve horizontally to the vertical axis. Read the value from the graph. In this case the value is 60.

Cumulative Frequency Graph for Time Spent on Homework



Therefore about 60 per cent of the pupils spend six hours or less on homework.

Example 4

What percentage of pupils spends more than 7 hours on homework?

Find the point marked '7 hours' on the horizontal axis and read the corresponding value from the graph. This shows that 70 % of pupils spend 7 hours or less on homework.

Cumulative frequency graph for time spent on homework



The question asked was how many pupils spend more than 7 hours per week on homework, therefore $100 - 70\% = 30\%$ spend more than 7 hours on homework.

70% of pupils spend 7 hours or less on homework per week.

So 30% of pupils spend more than 7 hours on homework per week.

Note: percentages are sometimes referred to as percentiles in this context. Percentiles are defined in the glossary.

Avoiding common errors

- Ensure the cumulative frequency graph is always plotted using the correct axes (cumulative frequency is always plotted on the vertical axis).
- Decide whether a question is asking for 'more than' or 'less than' a particular value (see example 4).
- Decide whether to read from the vertical axis to the horizontal axis or vice versa.

Formulae

A formula is a statement in words or symbols showing the relationship between 2 or more variables, eg, degrees Celsius and degrees Fahrenheit. Formulae are useful general statements which can be applied to different numbers on different occasions.

Example

You could write or use a formula to represent the total cost of a number of packs of magazines at £2 per pack and additional individual copies at 40p each. You would need to have a variable P for the number of packs of magazines, a variable N for the number of individual copies required, and a variable C for the cost in pounds.

So for any order of packs and individual copies, the total cost is found by:

$$(P \times £2) + (N \times 40p)$$

The cost of individual copies is in pence, but the cost of the packs and the total cost are both in pounds. When constructing a formula you need to make sure that all the units are the same. To keep them all in pounds, write 40p as £0.4. Then the formula is:

$$C = (P \times 2) + (N \times 0.4)$$

or

$$C = 2P + 0.4N$$

To find the total cost for an order of 7 packs and 3 individual copies, replace P and N with the appropriate numbers. In this case P is 7 and N is 3.

$$C = (7 \times 2) + (3 \times 0.4)$$

$$= 14 + 1.2$$

$$= 15.2$$

So the total cost is £15.20.

Many GCSE syllabuses are examined by testing several components which are said to be 'weighted'. For example, in GCSE science (1999-2000 syllabuses) the practical tasks are marked out of 30 and the score, when doubled, counts towards the final mark. So here, the practical tasks have double weighting. Then a score out of 3 for spelling, punctuation and grammar (SPaG) is included. A formula is applied to the pupils' marks to produce a final mark out of 63.

There are three variables: the practical mark, the SPaG mark, and the total mark.

Call the practical mark P, the SPaG mark S, and the total mark T.

The formula is:

$$T = (P \times 2) + S \text{ or } T = 2 P + S$$

1 pupil got 24 for the practical mark and 2 for the SPaG. So,

$$P = 24, S = 2$$

$$\text{and } T = (2 \times 24) + 2$$

$$= 50$$

So this pupil got a mark of 50 out of a possible 63.

Worked examples

Example 1

An examination consisted of 2 papers. Paper 1 is marked out of 80 with a SPaG mark out of 4 added. Paper 2 is marked out of 120, with a further SPaG mark out of 4 added. The final mark is obtained by adding together the 4 marks and giving this total as a percentage, rounded to the nearest whole number.

In Paper 1, a pupil gets 57 out of 80 and for SPaG 3 out of 4. In Paper 2, he gets 78 out of 120 and for SPaG 2 out of 4. What will his final mark be, expressed as a percentage?

There are 2 stages required to answer this question.

Stage 1

Paper 1 is marked out of 80 and the SPaG mark out of 4, so the maximum possible mark is 84.

Paper 2 is marked out of 120 and SPaG mark out of 4, so the maximum possible mark is 124.

The total maximum possible maximum mark from both papers is $84 + 124 = 208$.

The formula required to obtain candidates' final marks out of 208 is:

$$\text{Final mark} = (\text{mark} + \text{SPaG for paper 1}) + (\text{mark} + \text{SPaG for paper 2})$$

Inserting the pupil's marks into the formula gives:

$$\text{Final mark} = (57 + 3) + (78+2)$$

$$= 60 + 80$$

$$= 140$$

Therefore the pupil achieved 140 out of 208 or 140/208.

Stage 2

Convert 140/208 to a percentage.

$$140 \div 208 = 0.67307692 = 67.31\%$$

= 67% to the nearest whole number

Therefore the pupil's final mark is 67%.

Example 2

A teacher sets her pupils 3 tests in year 7. She weights the marks in the second test by 3 and those of the third test by 5. Each test is marked out of 100, so the total possible marks are $100 + 300 + 500 = 900$.

Call the marks gained in the first test T1, those in the second test T2, and in the third test T3.

The total mark T is found by the formula:

$$T = T1 + (T2 \times 3) + (T3 \times 5)$$

or

$$T = T1 + 3T2 + 5T3$$

A pupil gets 15 for the first test, 21 for the second test, and 58 for the third test.

$$T1 = 15, T2 = 21 \text{ and } T3 = 58$$

$$T = 15 + (21 \times 3) + (58 \times 5)$$

$$= 15 + 63 + 290$$

$$= 368$$

So his total mark is 368 out of 900.

To turn this into a percentage:

$$368 \div 900 = 0.4088$$

= 40.88 or 41% to the nearest whole number.

Avoiding common errors

Most common errors can be avoided by:

- ensuring calculations are carried out in the correct order (carry out multiplication and division before addition and subtraction);
- knowing that a symbol (usually a letter) is simply a shorthand way of writing a longer statement, eg (A) represents the marks out of 65 on test 1, (B) represents the number of books ordered; and
- ensuring that you know what the units are before you calculate, so you don't confuse pounds with pence, metres with centimetres, and so on.

Fractions, decimals and percentages

Fractions, decimals and percentages are all forms for expressing parts of a whole and each can be represented in any of the 3 forms. For example, the fraction $\frac{1}{2}$ can be represented as a decimal (0.5) or as a percentage (50 %).

Fractions express parts of a whole. Fractions are given in the form of a numerator and a denominator. The denominator indicates what size part is being expressed and the numerator indicates how many of those parts are involved. For example, in $\frac{6}{7}$ the denominator (7) indicates that each part is 1 seventh of the whole. The numerator (6) indicates that there are 6 of these parts.

To find $\frac{6}{7}$:

$$\frac{1}{7} \text{ of } 56 = 56 \div 7 = 8$$

$$\frac{6}{7} \text{ of } 56 = 8 \times 6 = 48$$

Converting fractions to decimals and vice versa

Converting a fraction to a decimal

A fraction can always be converted to a decimal by dividing the top (numerator) by the bottom (denominator).

So the fraction $\frac{60}{150}$ can be converted to a decimal by division: $60 \div 150$, or $6 \div 15$, or $2 \div 5$.

Each of these will give the same answer 0.4, because all 3 fractions have the same value, that is, they (are equivalent). The on-screen calculator can be used to carry out the division.

It is worthwhile remembering some common equivalents.

$$\frac{3}{4} = 0.75$$

$$\frac{1}{2} = 0.5$$

$$\frac{1}{4} = 0.25$$

Converting a decimal to a fraction

A decimal such as 0.7 can be written as a fraction by remembering that the first decimal represents tenths, the second hundredths and so on.

So $0.7 = \frac{7}{10}$, $0.46 = \frac{46}{100}$ and so on.

Sometimes the result of division gives more digits after the decimal point than are needed or are appropriate. In this case the number would be rounded up or down, remembering always to state the degree of accuracy that has been given. So an answer 63.472418 might be rounded up to 63.5 to 1 decimal place or to 63.47 to two decimal places.

Converting fractions and decimals to percentages and vice versa

Converting a percentage to a fraction

As a percentage represents parts out of a hundred, it is easy to convert a percentage to a fraction.

$$\begin{aligned}\text{So } 60\% &= \frac{60}{100} \\ &= \frac{30}{50} \text{ (by dividing top and bottom numbers by 2)} \\ &= \frac{3}{5} \text{ (by dividing top and bottom numbers by 10)}\end{aligned}$$

$$\begin{aligned}75\% &= \frac{75}{100} \\ &= \frac{3}{4} \text{ (by dividing top and bottom numbers by 25)}\end{aligned}$$

It is worthwhile remembering the more common percentages:

$$\begin{aligned}10\% &= \frac{1}{10} \text{ (divide by 10)} \\ 12.5\% &= \frac{1}{8} \text{ (divide by 8)} \\ 20\% &= \frac{1}{5} \text{ (divide by 5)} \\ 25\% &= \frac{1}{4} \text{ (divide by 4)} \\ 40\% &= \frac{2}{5} \text{ (divide by 5, multiply by 2)} \\ 50\% &= \frac{1}{2} \text{ (divide by 2)} \\ 75\% &= \frac{3}{4} \text{ (divide by 4, multiply by 3)} \\ 100\% &= 1 \text{ (divide by 1)}\end{aligned}$$

Converting a decimal to a percentage

It is easy to convert a decimal to a percentage, because, as already seen above, decimals can always be easily written with a denominator of a hundred.

$$\begin{aligned}0.5 &= \frac{1}{2} \\ &= \frac{50}{100} \\ &= 50\%\end{aligned}$$

$$\begin{aligned}0.37 &= \frac{37}{100} \\ &= 37\%\end{aligned}$$

$$\begin{aligned}
0.413 &= \frac{413}{1000} \\
&= \frac{41.3}{100} \\
&= 41.3\%
\end{aligned}$$

Converting a fraction to a percentage

Fractions with denominators of 10 or closely related to 10 can be converted to a percentage as follows.

$$\frac{7}{10} = \frac{70}{100} = 70\%$$

$$\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$$

$$\frac{9}{20} = \frac{45}{100} = 45\%$$

$$\frac{7}{25} = \frac{28}{100} = 28\%$$

Example

A pupil scores $\frac{71}{80}$ in a test; what percentage is this?

$71 \div 80$ using a calculator gives 88.75%.

For most fractions it is easier to convert them first into decimals (using a calculator if appropriate) and then convert that decimal into a percentage.

$$\frac{4}{9} = 4 \div 9 = 0.444444 = 44.4444\% = 44\% \text{ to the nearest whole number.}$$

$$\frac{13}{17} = 13 \div 17 = 0.7647058 = 76.47058\% = 76\% \text{ to the nearest whole number, or } 76.5\% \text{ to 1 decimal place.}$$

Some of the most common fractions, decimals and their percentage equivalents:

$$1\% = \frac{1}{100} = 0.01 \text{ (divide by 100)}$$

$$5\% = \frac{1}{20} = 0.05 \text{ (divide by 20)}$$

$$10\% = \frac{1}{10} = 0.1 \text{ (divide by 10)}$$

$$12.5\% = \frac{1}{8} = 0.125 \text{ (divide by 8)}$$

$$20\% = \frac{1}{5} = 0.2 \text{ (divide by 5)}$$

$$25\% = \frac{1}{4} = 0.25 \text{ (divide by 4)}$$

$$50\% = \frac{1}{2} = 0.5 \text{ (divide by 2)}$$

$$75\% = \frac{3}{4} = 0.75 \text{ (divide by 4, multiply by 3)}$$

You should learn all of these and be able to switch between equivalent forms. This knowledge could be particularly helpful when answering mental arithmetic questions.

Examples

1. A maths department ordered books to the value of £96. The supplier gave a discount of 12.5%. $\frac{1}{8} \times 96 = 12$

The saving is £12, so the books actually cost £84.

2. Three eighths of a class of 30 pupils have school dinners. What percentage do not have school dinners?

$\frac{3}{8} = 37.5\%$, so 62.5% do not have school dinners.

Simple percentage calculations

Some percentages are very easy to calculate both in your head and on a calculator. For example, it is always straightforward to find 10% of a quantity in your head by remembering that 10% is $\frac{1}{10}$. So, to find 10% of anything, all you need to do is divide by 10.

What is 10% of 632?

$$10\% \text{ of } 632 = 632 \div 10 = 63.2$$

Finding 10% can often help with finding other percentages in your head.

Examples

1. What is 30 % of 40?
 $10\% \text{ of } 40 = 40 \div 10 = 4$
 $30\% \text{ of } 40 = 4 \times 3 = 12$
2. What is 15 % of 30?
 $10\% \text{ of } 30 = 3$
 $5\% \text{ of } 30 = 3 \div 2 = 1.5$
 $15\% \text{ of } 30 = 3 + 1.5 = 4.5$

The other common percentages can be used in the same way to find answers in your head.

What is 75 % of 80p?

$$75\% \text{ of } 80 = \frac{3}{4} \text{ of } 80$$

$$\frac{1}{4} = 20$$

$$\frac{3}{4} \text{ of } 80 = 3 \times 20 = 60$$

Other more complex percentages can also be found easily by converting the percentages to a decimal, using a calculator if appropriate.

What is 73% of 84?

Convert 73% into a decimal in your head.

$$73\% = 0.73$$

$$0.73 \times 84 \text{ on the calculator, or by other means} = 61.32$$

You can check whether your answer is about right by estimation. For example 73% is about 75% which you know is $\frac{3}{4}$. So you know the answer will certainly be more than $\frac{1}{2}$ of 84 but less than 84.

Using percentages to compare data

Percentages are often useful when making comparisons between data.

Example

Total number of half days of unauthorised absences each term

	Term 1	Term 2	Term 3	Total number of absences
Year 9 (1998/99)	46	60	44	150
Year 9 (1997/98)	34	78	18	130

In which year was the percentage of term 2 unauthorised absences, out of total absences, the greater?

From the table, the term 2 absences for the year 1998/99 are 60 out of 150.

For the year 1997/8 the absences for term 2 were 78 out of a total absences of 130.

Convert both of these fractions to percentages. The first can be done mentally:

$$\begin{aligned} \frac{60}{150} &= \frac{6}{15} \\ &= \frac{2}{5} \\ &= \frac{4}{10} \\ &= 40\% \end{aligned}$$

For the second, convert the fraction first to a decimal, using a calculator if you need to, and then to a percentage.

$$\begin{aligned} \frac{78}{130} &= 78 \div 130 \\ &= 0.60 \\ &= 60\% \end{aligned}$$

The percentage of unauthorised absences in term 2 for 1997/98 = 60%.

The percentage of unauthorised absences in term 2 for 1998/99 = 40%.

This means that the percentage of term 2 unauthorised absences out of total absences was greater for 1997/98.

Worked examples

Example 1

A pupil obtained the following marks in 3 tests.

	Test 1	Test 2	Test 3
Actual mark	95	39	61
Possible mark	150	60	100

In which test did the pupil achieve the best result?

To answer this question using percentages, first convert each fraction to a decimal, using a calculator where appropriate:

$$\begin{aligned} \frac{95}{150} &= 95 \div 150 \approx 0.63 \text{ to 2 decimal places} \\ \frac{39}{60} &= 39 \div 60 \approx 0.65 \\ \frac{61}{100} &= 61 \div 100 \approx 0.61 \end{aligned}$$

Now convert each decimal result above to a percentage in your head.

$$0.63 = 63\%$$

$$0.65 = 65\%$$

$$0.61 = 61\%$$

65% is the highest percentage. The pupil achieved the best result in test 2.

Selecting the best method of comparing data is a matter of judgement. However, with data presented as in the question above, percentages would normally be used.

Example 2

Four schools had the following proportion of pupils receiving free school meals.

School	Proportion of pupils receiving free school meals
A	$\frac{1}{9}$
B	0.12
C	11%
D	11 pupils out of 110

Which school had the highest proportion of pupils receiving free school meals?

Converting all the proportions to percentages enables a comparison to be made easily.

The proportion of pupils having free school meals in school A is given as the fraction $\frac{1}{9}$.

$1 \div 9 = 0.11$ to two decimal places, or 11%

The proportion for school B is given as a decimal: $0.12 = 12\%$

The proportion for school C is given at 11%

The proportion for school D is given as 11 out of 110. This is the same as $\frac{11}{110}$. First convert this to a decimal:

$11 \div 110 = 0.1 = 10\%$

School A	School B	School C	School D
11%	12%	11%	10%

So school B had the highest proportion of pupils receiving free school meals because 12% is the highest percentage.

Example 3

The numbers of year 13 pupils who left school to enter employment at the end of 1998 and at the end of 1999 are given in the table below.

Destination	1998	1999
Employment	50	60
Total number of Year 13 pupils	525	540
Percentage entering employment	9.5%	11.1%

By how many percentage points had the number of pupils entering employment increased between 1998 and 1999?

Finding the increase in percentage points is just a matter of comparing the percentages.

The percentage entering employment in 1998 was 9.5% and in 1999 was 11.1%.

The increase was $11.1\% - 9.5\% = 1.6\%$

The increase was 1.6 percentage points.

By what percentage did the numbers entering employment increase between 1998 and 1999?

To find the percentage increase, we need to think what any increase is to be compared to. There was an actual increase of pupils entering employment of 10. Compare this increase with the original number of pupils to get a measure of the significance of the increase.

To calculate the percentage increase that 10 more pupils represents, it is necessary to find 10 as a percentage of the number of pupils in employment last year, that is, 10 as a percentage of 50.

$$\frac{10}{50} = 10 \div 50 = 0.2 = 20\%$$

So, the number entering employment increased by 20% between 1998 and 1999.

Note: When entering answers which are fractions, eg 5 eighths, the answer will appear as $\frac{5}{8}$.

Avoiding common errors

Most errors can be avoided by:

- memorising equivalent values for commonly used percentages, fractions and decimals;
- recognising the equivalence between decimals, fractions and percentages, for example, $\frac{35}{100}$, 0.35 and 35%;
- knowing how to convert from fraction into decimals and percentages into another form and back again;
- simplifying fractions to ease understanding and use; and
- clearly understanding the difference between a percentage increase or decrease and a percentage point increase or decrease.

Line graphs

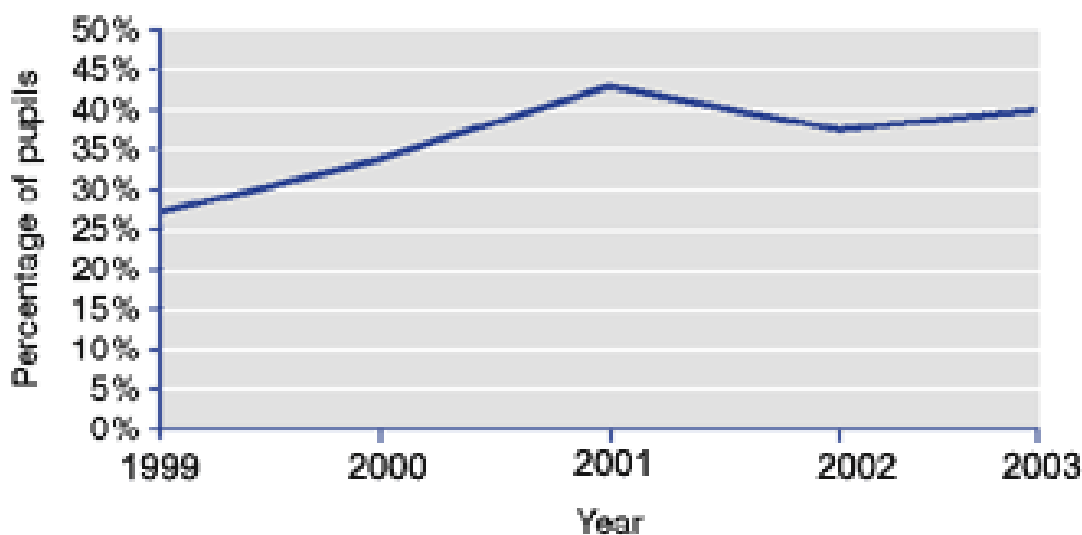
A line graph is a visual representation of two sets of related data. It is the name given to a graph where the individual points are joined by a line or lines.

Example

The percentage of pupils in a school achieving level 5 in the end of key stage 3 assessments was recorded in the table below.

Percentage of pupils achieving level 5 in the end of key stage 3 assessments

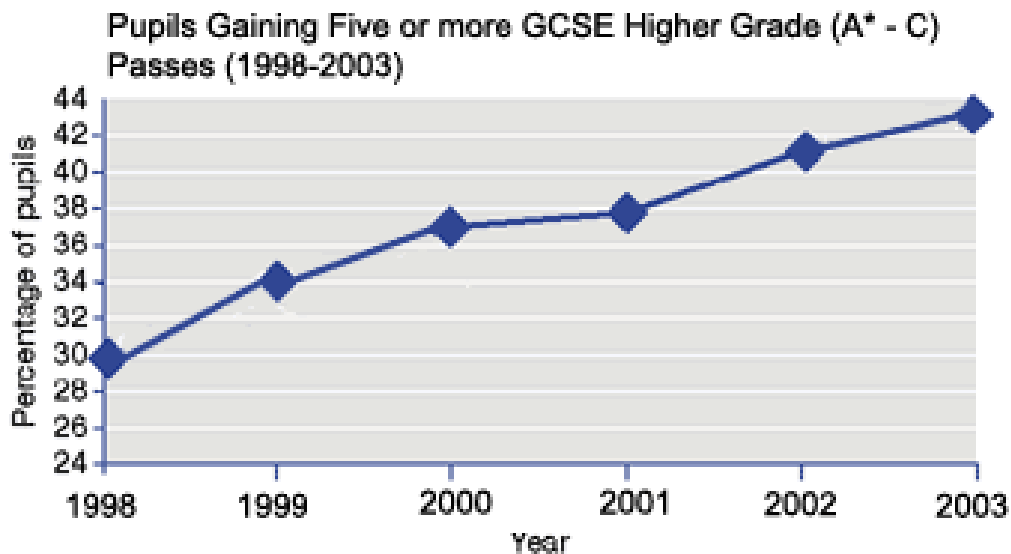
Year	1999	2000	2001	2002	2003
% of pupils achieving level 5	27%	34%	43%	37%	40%



The table and the line graph represent the same data. The graph provides a visual representation of the changes in the school's performance over a period of time, but the points relating to the years would not normally be joined up; they have been here to help to show the trend.

Worked examples

Example 1



By how many percentage points did GCSE achievement of 5 or more (A* to C) passes increase from 1999 to 2003?

Find 1999 on the 'year' axis, move vertically upwards to the graph line, then horizontally across to the 'percentage of pupils' axis. The corresponding value is 34%.

Now do the same for 2003. The value on the 'percentage of pupils' axis for 2003 is 43%.

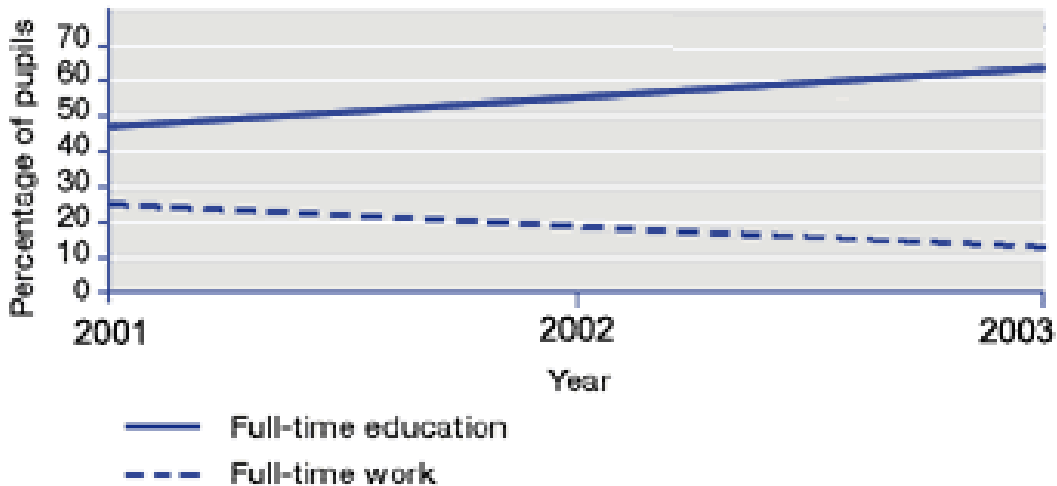
Subtract the 1999 figure from the 2003 figure to obtain the percentage point increase:

$$43\% - 34\% = 9\%.$$

So GCSE achievement of 5 or more (A* to C) passes from 1999 to 2003 increased by 9 percentage points.

Example 2

16-17 Year-old Pupils Progressing to Full-time Education/Work



What is the change, between 2001 and 2003, in the percentage of 16 to 17 year olds in full time education and full time work?

Find 2001 on the 'year' axis and the corresponding percentage of pupils in full time work on the vertical axis. It is 22%.

Find the percentage of pupils in full time education for the year 2001 on the vertical axis. It is 47%.

Repeat the process for the year 2003. The figures are 10% for full time work and 64% for full-time education.

So the percentage of 16-17 year-olds moving into full time work declined from 22% in 2001 to 10% in 2003, which is a fall of 12%.

The percentage in full time education increased from 47% in 2001 to 64% in 2003, which is an increase of 17%.

Avoiding common errors

Most common errors can be avoided by:

- reading the title to check what the graph represents;
- checking where the axes start;
- checking which units are being used on each axis; and
- checking the scale on each axis.

Pie-charts

A pie chart is a way of illustrating information by using sectors of a circle to represent parts of the whole.

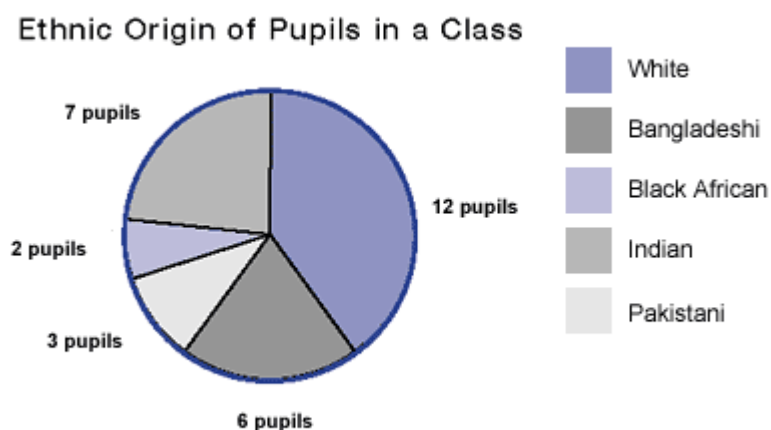
Example

A newly qualified teacher (NQT) was given the following information about the ethnic origins of the pupils in a class.

Ethnic origin	No. of pupils
White	12
Indian	7
Black African	2
Pakistani	3
Bangladeshi	6
Total	30

The 30 pupils in the class are classified into 5 different ethnic origins. The whole pie chart represents the class of 30 pupils, and the 5 sectors represent 12, 7, 2, 3 and 6 pupils, as shown in the table.

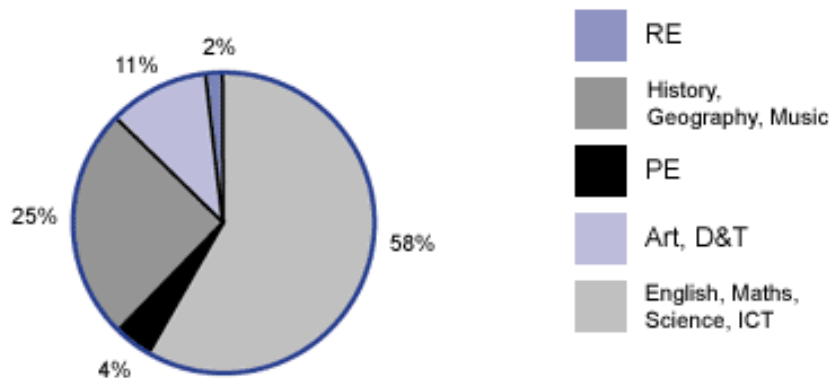
The pie chart has a legend (key) indicating what it represents.



Worked examples

The pie chart below summarises the amount of time spent on various areas of the key stage 2 curriculum in a primary school.

Time Spent on Key Stage 2 Curriculum



Example 1

The total time spent each week on the various areas of the curriculum is 26 hours. How much time is spent on history, geography and music?

The pie chart represents 26 hours.

Look at the pie chart and note that the total time spent on history, geography and music is 25%.

25% is $\frac{1}{4}$, so 25% of the total time is $26 \div 4 = 6.5$ hours. 6.5 hours = 6 hours and 30 minutes. So 6 hours and 30 minutes is spent on history, geography and music each week.

Example 2

The total time spent each week on the various areas of the curriculum is 26 hours. How much time is spent each week on English, mathematics, science and ICT together?

The pie chart represents 26 hours.

Look at the pie chart and note that 58% of the total time was spent on English, mathematics, science and ICT.

58% can be converted to a decimal.

$$58\% = 58 \div 100 = 0.58$$

Using a calculator, $0.58 \times 26 = 15.08$

To find how many minutes the 0.08 represents in the answer, 15.08 hours, you can calculate: $0.08 \times 60 \text{ minutes} = 4.8 \text{ minutes}$

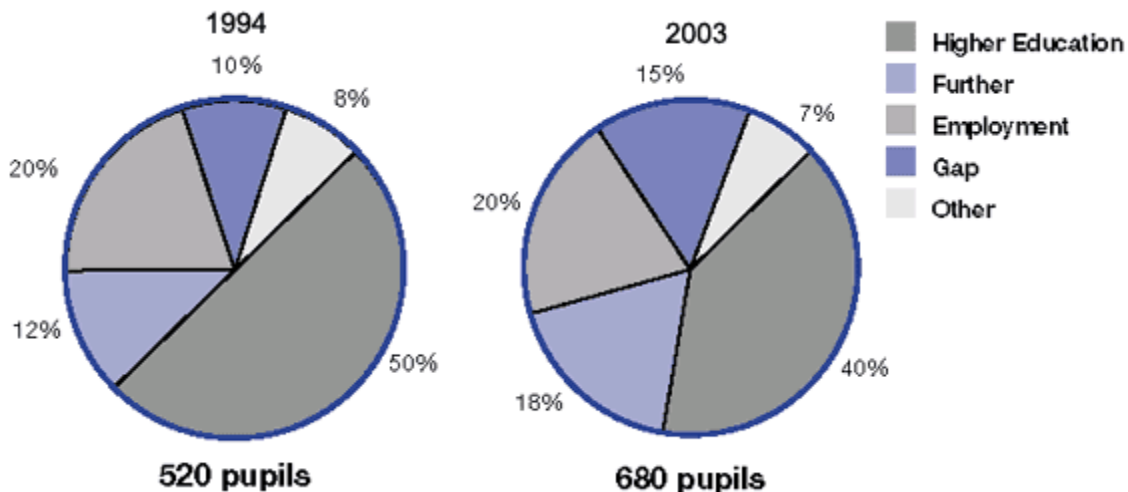
So to the nearest minute, 15 hours and 5 minutes are spent each week on English, mathematics, science and ICT together.

Using pie charts to compare data

Pie charts are often used to show comparisons between 2 sets of data.

Worked example

The following pie charts show an analysis of the destinations of pupils leaving a sixth form college.



Which of the following statements are true?

1. More pupils went into higher education in 1994 than in 2003
2. 136 pupils went into employment in 2003
3. More pupils decided on a gap year in 2003 than in 1994.

Statement 1

In 1994, 50% of pupils went into higher education and in 2003 only 40 %, but the number of pupils is different.

$50 \% \text{ of } 520 = \frac{1}{2} \times 520 = 260 \text{ pupils}$

$40 \% \text{ of } 680 = \frac{40}{100} \times 680 = 272 \text{ pupils}$

There were more pupils in 2003, so statement 1 is false.

Statement 2

In 2003, 20% of pupils went into employment.

$20\% \text{ of } 680 = 20 \div 100 \times 680 = 136 \text{ pupils}$

Statement 2 is true.

Statement 3

In 1994, 10 % of pupils decided on a gap year.

$1/100 \times 520 = 52 \text{ pupils}$

In 2003, 15 % of pupils decided on a gap year.

$15/100 \times 680 = 102 \text{ pupils}$

Statement 3 is true.

Proportion

A proportion is used to describe the relationship of some part of a whole to the whole itself and is usually given as a fraction.

Example

In a class there are 20 girls and 10 boys. The ratio of girls to boys is 20:10, or 2:1, and the proportion of girls in the class is 20 out of 30 or $\frac{20}{30} = \frac{2}{3}$.

Worked examples

Example 1

An infant school has 180 pupils, of whom 35 go home for lunch. What proportion of pupils has lunch at school?

35 out of 180 go home.

So $180 - 35 = 145$ stay at school for lunch.

As a proportion this is represented as 145 out of 180

$\frac{145}{180} = \frac{29}{36}$ (dividing top and bottom by 5)

So the proportion that stays at school for lunch is $\frac{29}{36}$.

Example 2

Three schools have the same proportion of pupils receiving free school meals, $\frac{8}{75}$. If the numbers of pupils receiving free school meals in each of the 3 schools are 16, 40 and 56 respectively, how many pupils in each school do not receive free school meals?

The proportion of pupils receiving free school meals is 8 out of every 75, or $\frac{8}{75}$.

To work out the number of pupils in each of the 3 schools who do not receive free school meals, you need to find out the total number of pupils in each school. You can do this using the proportion $\frac{8}{75}$ by relating the 16, 40, 56 to the 8 out of every 75 proportion given.

8 out of 75 is 16 out of 150 (the 8 has been doubled. To maintain the same proportion the 75 must also be doubled, giving 150)

is 40 out of 375 (the 8 has been multiplied by 5; $75 \times 5 = 375$)

is 56 out of 525 (the 8 has been multiplied by 7; $75 \times 7 = 525$)

So the total number of pupils in each school is 150, 375 and 525 respectively.

To find the numbers of pupils who do not have free school meals in each school, simply subtract the number of pupils getting free school meals from the total number of pupils.

School 1

150 pupils of which 16 receive free school meals, so $150 - 16 = 134$ pupils do not receive free school meals.

School 2

375 pupils of which 40 receive free school meals, so $375 - 40 = 335$ pupils do not receive free school meals.

School 3

525 pupils of which 56 receive free school meals, so $525 - 56 = 469$ pupils do not receive free school meals.

Avoiding common errors

Most common errors can be avoided by remembering that proportion can be scaled by multiplying or dividing both parts by the same number.

Range

Range is a measure of the spread of data. It is the difference between the largest and the smallest values. It is used, eg, in looking at the results of a test, to see what range of the available marks has been achieved. To find the range of a set of data, take the smallest value from the largest value.

Worked examples

Example 1

In a set of tests, 2 pupils scored the following marks out of 10:

John: 5, 6, 4, 5, 5, 6, 5, 7

Sally: 3, 2, 3, 6, 5, 8, 6, 9

The lowest mark John scored is 4 and the highest mark he scored is 7, so the range of John's marks is 3 (from 4 to 7).

Sally's marks are between 2 and 9 making the range for Sally's marks 7.

From this we can see that Sally's marks are more widely spread than John's marks. John's marks are fairly consistent, whereas Sally obtains some high and some low marks.

Example 2

The marks scored by a group of pupils in an end of term test marked out of 80 are as follows.

37, 45, 53, 61, 70, 50, 48, 29, 52, 59

The teacher was interested in the spread of marks. What is the range of the set of marks?

The lowest mark is 29

The highest mark is 70

The difference: $70 - 29 = 41$

The range of marks is 41. This result shows wide variation in the group's performance on this test.

Example 3



The scatter graph shows the percentage scores of a group of pupils in test A and test B. Each dot on the graph represents 1 pupil. So pupil X scored 58 in test A and 48 in test B. Pupils' scores might be plotted on a scatter graph to see whether there is a correlation between the marks on the 2 tests. It shows that pupil P scored well in both tests, but that pupil M scored well on test A but badly on test B. The range helps in the analyses of tests to see to what extent they distinguish between the high achieving and low achieving pupils.

Which test (A or B) had the greater range?

To find the range of marks in test A, look along the x axis (horizontal) and read off the first and last values.

The lowest percentage score for test A is 49.

The highest percentage score for test A is 80.

The range of percentage scores for test A is $80 - 49 = 31$.

To find the range of scores in for test B, look up the y axis (vertical) and read off the first and last values.

The lowest percentage score for test B is 10.

The highest percentage score for test B is 81.

The range of percentage scores for test B is $81 - 10 = 71$.

Test A percentage range is 31.

Test B percentage range is 71.

The greater range is test B.

So in test B, a wider range of percentage scores were obtained.

Ratio

A ratio is a comparison between 2 quantities.

Example 1

If there are 20 girls and 10 boys in a class, the ratio of girls to boys is 20 to 10. This is written as 20:10. Ratios, like fractions, can be simplified, so 20:10 could be written as 2:1. In this example, the 2 'quantities' being compared are girls and boys. 20 girls are compared with 10 boys. This comparison is summarised by using the colon symbol (:).

The quantities are always compared in the order of the statement. In the example, girls are compared with boys. So the ratio of girls to boys is 20:10. If the boys are compared with the girls, the ratio is 10:20, or 1:2.

The ratio 20:10 may be simplified to 2:1. The ratio 2:1 expresses the fact that there are 2 girls for every 1 boy in the class. The original ratio 20:10 not only expresses the fact that there are 20 girls for every 10 boys in the class, but also the fact that the class contains 30 pupils (see the page on proportion).

In the example above, like quantities are being compared as both girls and boys are pupils. Ratios must always compare 2 values of the same type of data. So if 6 hours are to be compared with 1 day, the ratio is 6:24 because there are 24 hours in a day.

Example 2

A school has 360 key stage 3 (KS3) pupils and 200 key stage 4 (KS4) pupils. Show this information as a ratio.

The ratio which compares the number of KS3 pupils with the number of KS4 pupils is:

$$360:200$$

This can be simplified by dividing by 10 to:

$$36:20$$

This ratio shows that for every 36 KS3 pupils there are 20 KS4 pupils.

Simplifying this further by dividing each number by 4 gives:

$$9:5$$

The ratio 9:5 states that for every 9 KS3 pupils there are 5 KS4 pupils.

The ratio of KS3 pupils to KS4 pupils is 9:5.

Example 3

In a 25 hour week of lessons a pupil has 5 hours of science and 2.5 hours of design and technology (D&T). Find the ratio of D&T hours to science hours and the ratio of science hours to the total lesson hours. Express both as ratios in their lowest form.

The ratio of D&T hours to science hours is:

2.5 to 5 hours or 2.5:5, which is 1:2 in its simplest form. So for every 2.5 hours of D&T, there were 5 hours of science.

The ratio of science hours to total lesson hours is: 5 to 25 or 5:25, which is 1:5 in its lowest form.

Avoiding common errors

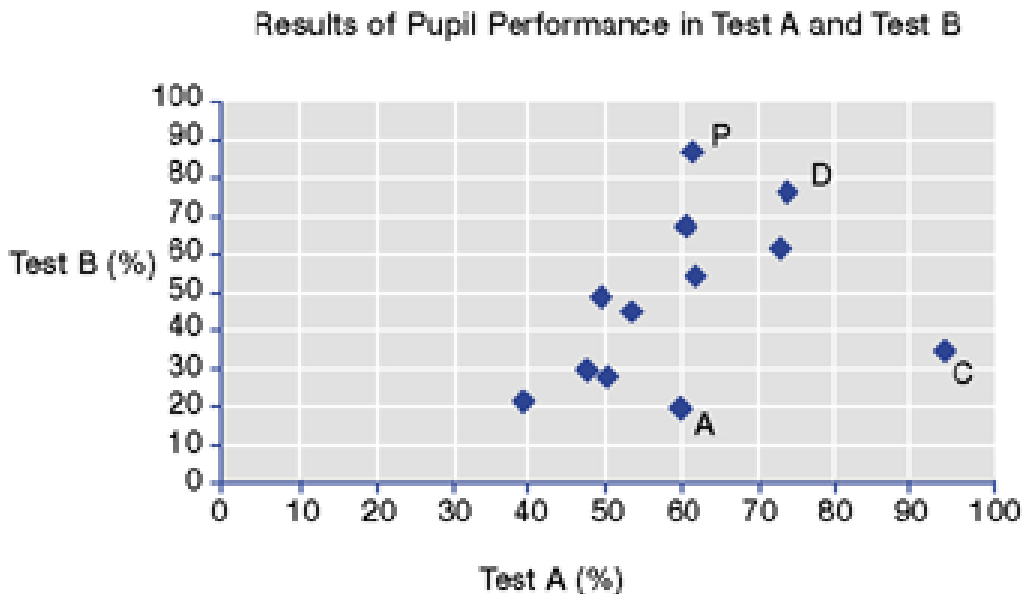
Most common errors can be avoided by simplifying the ratio and ensuring the numbers in the ratio are in the same order as the required comparison.

Scatter graphs

A scatter graph is a statistical diagram drawn to compare 2 sets of data. It can be used to look for connections or a correlation between the 2 sets of data.

Example

A class took 2 tests. Test A was given early in the course and test B towards the end. The comparative results of these tests are given in the scatter graph below.



Each symbol on the graph shows the scores achieved in both tests by each of the pupils. So, for example, pupil A scored 60% on test A but only 20% on test B.

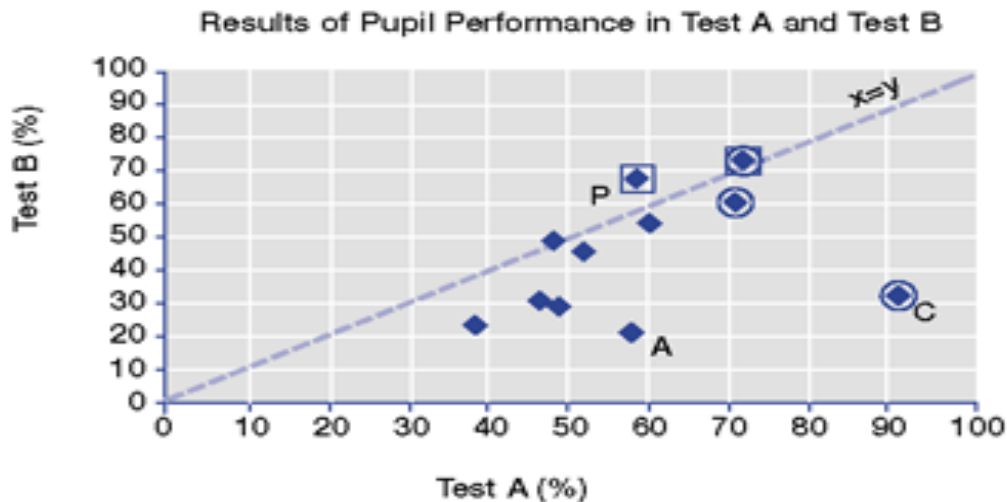
Pupil C achieved the top mark in test A.

Pupil P achieved the top mark in test B.

Pupil marked D came second in both tests.

Making comparisons

To find out whether the pupils generally did better in 1 test or the other, use a ruler or straight edge to draw a line joining marks that are the same on both tests, for example 0 for both, 50 for both, 70 for both, etc.



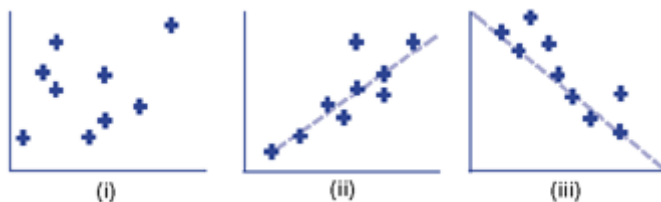
The 2 students above the line achieved higher marks on test B than in test A.

The 8 students below the line achieved higher marks on test A than on test B.

An important use of scatter graphs is to show how 1 set of results relates to another.

If the points on a scatter graph appear to be randomly scattered (see figure (i) below) there is unlikely to be any correlation between the 2 sets of data being measured.

If they form a more regular pattern (as in figure (ii) or figure (iii)), there is likely to be a correlation between them.



The scatter graph for the results from test A and test B, discussed earlier, is similar to figure (ii) and shows a correlation.

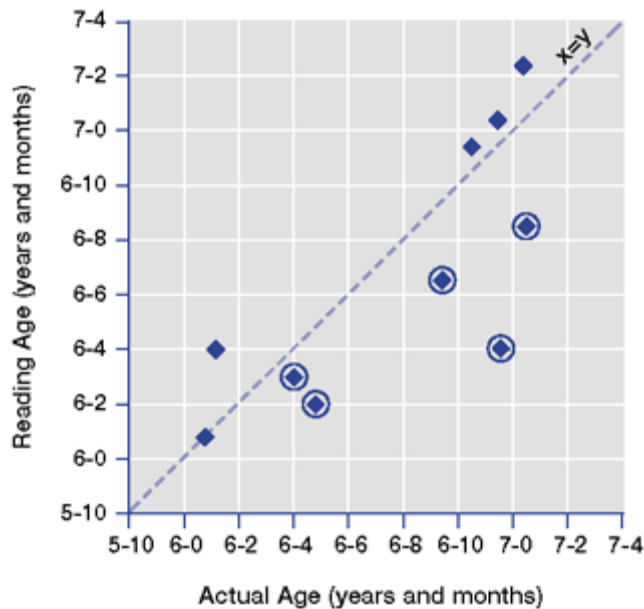
This suggests that test A and test B have a correlation because the pupils performed similarly in both tests. So the tests are likely to be based on the same subject or related subjects and set at a similar level.

The scatter graph enables you to identify particular pupils for whom action might be needed. For example, the reason why C achieved a high mark in test A but a low mark in test B might be investigated.

Worked examples

Example 1

The scatter graph below compares the reading age and the actual age of a group of pupils.



How many pupils have a reading age below their actual age?

To find the answer to this question, it is helpful to draw a line (as shown) through any pair of points that represent the same ages on both scales.

For example, (5-10, 5-10) and (6-8, 6-8) or (7-4, 7-4).

The 5 pupils whose marks are circled have a reading age that is clearly below their actual age.

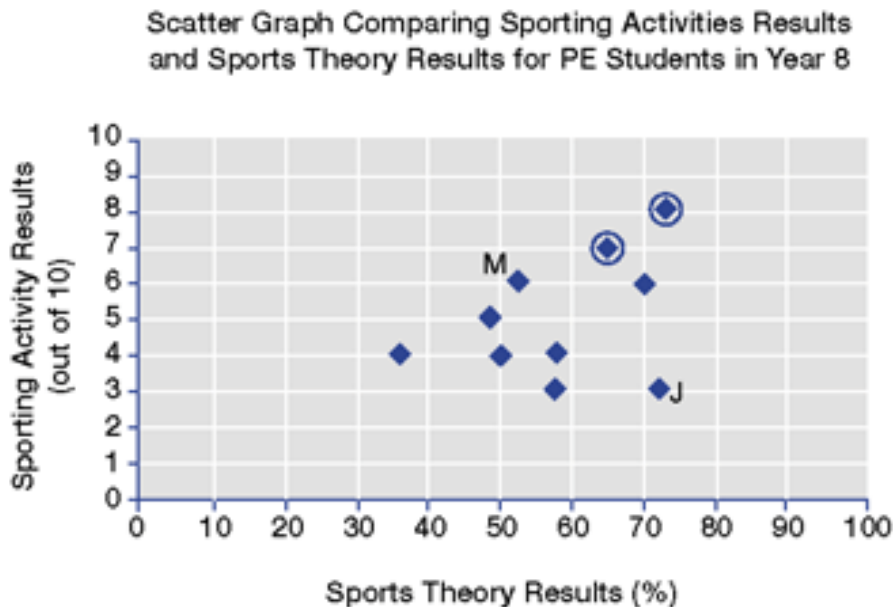
How old is the pupil who has a reading age the same as their actual age?

To find the answer to this question, look at the line you have drawn, joining (5-10, 5-10) and (7-4, 7-4) and find the symbol which lies on it.

The pupil is aged 6 years and 1 month.

Example 2

The PE department has been asked to show whether students in year 8 who do well in sporting activities do equally well in the sports theory part of the course and vice versa.



The students with the higher sporting activity results tend to get higher sports theory results. Those with the top 2 sporting activity marks indicated by circles also are among the top 4 theory results.

However, 1 student, M, achieved quite a high sporting activity result but came seventh out of 10 in sports theory. Conversely, J achieved quite a high mark in sports theory but the lowest mark in sporting activity.

Avoiding common errors

Most common errors can be avoided by:

- using a ruler or straight edge to move across the graph to help locate clusters of points;
- drawing horizontal, vertical or 45 degree lines to break up the graph to allow groups of points to be located more easily; and
- remembering that each point represents 1 person.

Time

In order to calculate answers to questions involving time, you must know which units are being used. In questions referring to a week, please note that it is a school week of 5 days.

When working out children's reading ages it is necessary to consider time given in years and months. A child may have an actual age of 8 years and 4 months, but a reading age of 8 years and 10 months. Another child of the same age might have a reading age of 7 years and 9 months. Reading ages are presented in several forms and 8 years 6 months, 8.6 or 8-6 are all common. If the form 8.6 is used, it has to be remembered that here the point is not a decimal point, just a means of separating months and years.

Example 1

A junior school is putting on entertainment. If each of the 4 classes is allowed 25 minutes and the entertainment starts at 2.15pm, when is it expected to finish?

If each class were allowed half an hour, the entertainment would finish 2 hours after the start, therefore 4.15pm.

However, each class has 25 minutes, which is 5 minutes less than half an hour. So 4 classes would take 4×5 minutes less, ie 20 minutes less.

The entertainment should therefore finish 20 minutes before 4.15pm. 15 minutes before 4.15pm is 4.00pm, 20 minutes before 4.15pm is 3.55pm.

An alternative method would be to calculate:

$25 \text{ minutes} \times 4 = 100 \text{ minutes} = 1 \text{ hour } 40 \text{ minutes}$

So the 4 classes take 1 hour 40 minutes.

1 hour 40 minutes after 2.15pm is 3.55pm.

Example 2

A parent comes in to a school for 1 and a half hours to hear a group of children reading. If there are 10 children in the class, can she give them all 10 minutes each?

One and a half hours is $60 + 30 = 90$ minutes.

To give 10 children 10 minutes each would take 100 minutes. So she cannot give each of them 10 minutes.

Avoiding common errors

Most common errors can be avoided by:

- checking the units used in the question;
- not using units of time as if they were decimals - for example, when referring to reading ages 7.5 does not mean $7\frac{1}{2}$ years (7 years 6 months), but 7 years 5 months. To avoid confusion, the notation 7-5 is often used;
- being clear about am and pm on the 12-hour clock, and
- taking care with times between 12.00 and 24.00 on the 24-hour clock.

Two way tables

Two way tables are designed to allow comparisons between 1 set of data and 2 others. For example, results for GCSE geography and religious education can be compared with the number of pupils obtaining the same grade in GCSE English language.

Example

Tables were prepared to show the relationship between GCSE English language grades and GCSE geography and religious education grades.

	GCSE geography						
	GCSE grade	A*-A	B-C	D-E	F-G	U-X	Total
GCSE English language	A*-A	10	6	1			17
	B-C	3	30	16	2		51
	D-E		3	19	7	3	32
	F-G						
	U-X						
	Total	13	39	36	9	3	100

	GCSE religious education						
	GCSE grade	A*- A	B-C	D-E	F-G	U-X	Total
GCSE English language	A*-A	6	5				11
	B-C	5	11	1			17
	D-E		12	14	1	1	28
	F-G		1	1	2		4
	U-X						
	Total	11	29	16	3	1	60

Indicate all the true statements:

1. The grade A* to C pass rate for GCSE religious education was exactly 10% higher than the grade A* to C pass rate for geography
2. Of the pupils taking GCSE geography, more than half achieved grade C and above in GCSE English language
3. Of the pupils taking GCSE religious education, one third did not achieve grade C or above.

Statement 1

Looking at the second table, the religious education results are given in columns. The grades A* to C pass rate for GCSE religious education is $11 + 29 = 40$. The total number of pupils taking religious education is 60.

The percentage of pupils who gained grades A* to C was:

$$\frac{40}{60} \times 100\% = 66.7\%$$

The grades A* to C pass rate for GCSE geography from the upper table, again shown in the columns, is $13 + 39 = 52$. The total number of pupils taking GCSE geography is 100, so the percentage is 52 %.

Statement 1 is therefore false, as $66.7 - 52$ is not 10.

Statement 2

Looking at the top table, the results for English language are given in rows. Looking across the top rows, 17 pupils gained grades A* to A for English language and 51 pupils gained grades B-C. Therefore 68 pupils gained grade C and above in English language. 100 pupils took geography and English language.

Therefore statement 2 is true, as 68 is more than half of 100.

Statement 3

Looking at the columns in the second table, the total number of pupils gaining grades A* to A and grades B to C is $11 + 29 = 40$. A total of 60 pupils took religious education, so 20 pupils did not achieve a grade C or above. This is the same as $\frac{1}{3}$, so statement 3 is true.

Avoiding common errors

Most common errors can be avoided by:

- using the correct totals;
- using the correct table;
- remembering that the totals are always on the opposite side of a table to the category heading.

Weighting

Example 1

In a GCSE examination which has 3 components (A, B and C), the raw marks for component A are weighted twice as much as the raw marks for component B, and those for component C are weighted half as much as the raw marks for component B. Thus, marks for A would be doubled and those for C halved before totalling, giving the formula:

$$\text{Final mark} = (2 \times A) + B + \frac{1}{2} C$$

Example 2

A pupil achieved the following marks in tests A, B and C.

Test	A	B	C
Raw mark	68	28	5

The pupil's weighted score was calculated using the following formula:

Weighted score =	$\frac{(A \times 60)}{100}$	+	$\frac{(B \times 30)}{80}$	+C
-------------------------	-----------------------------	---	----------------------------	----

What was the pupil's weighted score? Give your answer to the nearest whole number.

Weighted score =	$\frac{(68 \times 60)}{100}$	+	$\frac{(28 \times 30)}{80}$	+5
	$= 40.8 + 10.5 + 5 = 56.3$			

Example 3

A GCSE examination consists of 2 papers. Paper 1 has a maximum raw mark of 40 and a final weighting of 30%. Paper 2 has a maximum raw mark of 160 and a final weighting of 70%, as shown in the table:

	Maximum raw mark	Weighting %
Paper 1	40	30
Paper 2	160	70

Find the final weighted mark for a pupil who gained 27 marks in paper 1 and 93 marks on paper 2.

For paper 1: the score of 27 out of 40 is equivalent to a weighted percentage of

$$\frac{27}{40} \times 30\% = 20.25\%, \text{ rounded to } 20.3\%$$

For paper 2: the score of 93 out of 160 is equivalent to a weighted percentage of

$$\frac{93}{160} \times 70\% = 40.68\%, \text{ rounded to } 40.7\%$$

So the final weighted mark is $20.3\% + 40.7\% = 61\%$